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Dividing the equation $-2x^7 - x^5 + 10x^3 - 3x = 0$ by $x = 0$, we eliminate that root, and get after reduction, $x^6 + \frac{1}{2}x^4 - 5x^2 + \frac{3}{2} = 0$. Let $x^2 = v$. Then $v^3 + \frac{1}{2}v^2 - 5v + 1.5 = 0$. By Sturm's Theorem, there are three real roots, viz., one between -2 and -3 ; one between $+1$ and $+2$; and one between 0 and $+1$. By the original conditions of the problem, x can never be greater than 1 . Then by Horner's method $v = x^2 = .316336$, which is a trifle too large, but a very close approximation. Therefore $x = \pm .562437$. This is the abscissa of the point on the involute. It gives the function a minimum value. Radius of curvature $= .770961$. Hence the arc $\mu\nu$ or $\mu\lambda$, its equal, is convex to axis of abscissas, and $= 1 - .770961 = .229039$.

To find νi and Ci : i is a multiple point where the two branches intersect on the axis $C\mu$. We must now introduce the equation of the locus of the centres of the osculating circles. Let a represent the abscissa and b the ordinate of the centre of any osculatrix referred to the same origin that we have been using, at C :

$$y - b = -\frac{dx^2 + dy^2}{d^2y}; \therefore b = y - \frac{1 - x^2 + x^4}{1 + x^2}, a = -\frac{2x^5 - x^3}{1 - x^4}.$$

At the point i , $a = 0$, $\therefore 2x^5 - x^3 = 0$. Three of the roots are zero, $\therefore 2x^2 = 1$, $x = \pm \sqrt{\frac{1}{2}}$ and $x^2 = \frac{1}{2}$. Remembering that $y = \frac{1}{2} \log(1 - x^2)$, we find $b = \frac{1}{2} \log \frac{1}{2} - \frac{1}{2} = Ci = .8465736$.

To get νi , substitute for x^2 its value $\frac{1}{2}$ in the equation of the radius of curvature and we find for the point i , R. C. $= \sqrt{\frac{3}{4}} = \cos 30^\circ = .866025$. Subtracting from this the minimum value $.770961$, we get $\nu i = .095064$.

Co-ordinates of ν . The abscissa of the corresponding point on the involute, as given above, is $x = .562437$; \therefore by substituting, $a = .0726197$; $b = .785534$.

Length of arc $\nu\tau$ or $\lambda\pi$. In this case $a = -1$, $\therefore -\frac{2x^5 - x^3}{1 - x^4} = -1$; whence $x = .886$. Radius of curvature, at τ and π , $= 1.974699$. Subtracting as before the minimum value, $.770961$, we have $\nu\tau = 1.203738$, $i\tau = \nu\tau - \nu i = 1.108674$. Also $b = D\tau = 1.234227$.

Tangents, Normals &c. The tangent of the angle made by the tangent line to the evolute with the axis of ordinates is equal to $\frac{da}{db} = \frac{x}{1 - x^2}$, as may be seen by differentiating, reducing and dividing. The tangent of the angle made by the tangent to the involute $= -(1 - x^2) \div x$, therefore the tangent of the angle made by the normal to the evolute $= x \div (1 - x^2)$. Hence the tangent to the evolute is normal to the involute. This is a particular case of the general principle, and tests the accuracy of our calculations.

At the point ν , where the radius of curvature is a minimum, $x=.662437$. Therefore the tangent passing through this point and common to the two branches of the evolute, makes with the axis $C\mu$ an angle $= 39^{\circ}26'16''$.

The subnormal of the involute $= x(dx \div dy) = 1 - x^2$. For $x=0$, sub-normal $= 1 =$ radius of curvature. For $x=1$, sub-normal $= 0$, which shows that the normal is perpendicular to axis of ordinates at limit.

The normal $= \sqrt{1-x^2+x^4}$. For $x=0$, normal $= 1$; for $x=1$, nor. $= 1$.

Scholium. The discussion of these two correlated lines, the curve of logarithmic sines and its evolute, suggests an interesting conception of curvature, which may be made the basis of classification. The radius of curvature may be of constant or variable length.

(a). If the radius be constant, we have the circle.

(b). The radius may vary between finite limits, as in the ellipse, where the limits are $b^2 \div a$ and $a^2 \div b$; a and b denoting the semi-major and semi-minor axes.

(c). One limit may be zero, and the other a finite quantity, as in the cycloid. In this case the initial point of the evolute is on the involute. The radius of curvature equals twice the normal. Hence the limits are 0 and twice the diameter of the generating circle.

(d). One limit may be finite, and the other infinitely large, as in the spiral of Archimedes, the parabola, and the hyperbola, the logarithmic curve and the curve of logarithmic sines. The peculiarity of the last mentioned is that the radius of curvature does not increase steadily but at first diminishes until it reaches a minimum and then goes on increasing to infinity.

To these may be added the Lemniscate of Bernoulli and the Folium Cartesii.

(e). One limit may be zero, and the other ∞ , as in the logarithmic spiral. Here again the initial point of the evolute is on the involute, just as in the cycloid, and it is noteworthy that, in both cases, the evolutes are curves precisely equal to their involutes, and differing only in position.

Another needful element of the conception is that the radius of curvature revolves through a greater or a smaller arc. In the circle the ellipse, and if we make a re-entrant curve of two cycloids, or four curves of pursuit, in their cases also, radius of curvature revolves through 360° . In the parabola, and the curve of logarithmic sines, through 180° . In the hyperbola, the arc equal 180° less the angle which the asymptotes make with each other. In the spirals the arc is unlimited.